A. Simple problems

The suggested time allocated for **Question 1 to 4 is 12 minutes**.

1. Solve each of the following equations.

(a) $3(x-2)(x+1)=0$

$x-2=0$ or $x+1=0$
$x=2$ or $x=-1$

(b) $2(x-1)^2-6=0$ $\quad 2x^2-4x-4$

$(x-1)^2-3=0$
$(x-1)^2=3$
$x-1=\pm\sqrt{3}$
$x=\sqrt{3}+1$ or $x=-\sqrt{3}+1$

$\frac{4\pm\sqrt{16+32}}{4}$

(c) $3x^2-2x-1=0$

$(3x+1)(x-1)=0$
$x=-\frac{1}{3}$ or $x=1$

2. Find the remainder when $f(x)=x^3+x^2+3x+4$ is divided by $(x-1)$.

By Remainder theorem,
$f(1)=(1)^3+(1)^2+3(1)+4$
$=9$ ✓

3. If $f(x)=x^4-2x^2+k$ is divisible by $(x+2)$, find k.

By Remainder theorem,
$f(-2)=0$
□ $(-2)^4-2(-2)^2+k=0$
$16-8+k=0$
$k=-8$ ✓

4. If the function of a parabola is $y=2(x+1)^2-5$, find the vertex of the parabola.

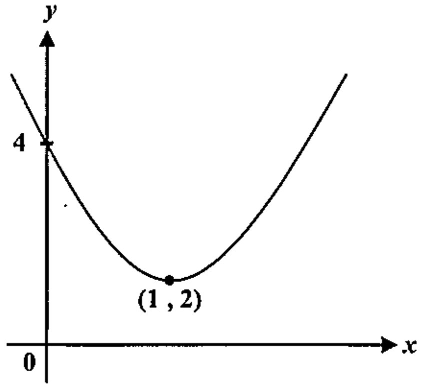
the vertex = (-1 , -5)

B. More complex problems

The suggested time allocated for **<u>Question 5 to 7 is 18 minutes</u>**.

5. Find the functions of each of the following parabolas.

(a)



where (1,2) is the **vertex** of the parabola.

Let the function be $\mathbf{y = a(x - h)^2 + k}$

At the vertex,

$y = a(x - 1)^2 + 2$

At the y-intercept,

$4 = a(0 - 1)^2 + 2$

$4 = a + 2$

$\underline{a = 2}$

The function is:

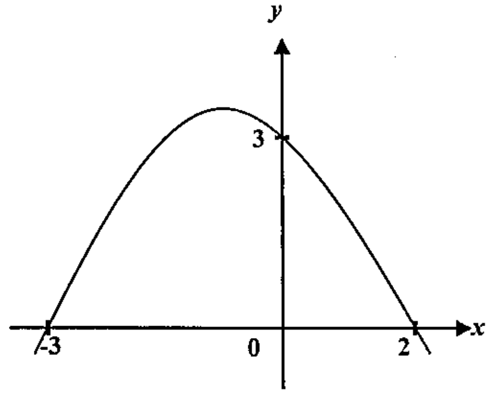
$\underline{y = 2(x - 1)^2 + 2}$

$y = 2(x^2 - 2x + 1) + 2$

$y = 2x^2 \cancel{+} - 4x + 2 + 2$

$y = 2x^2 \cancel{+} - 4x + 4$

(b)



Let the function be $\mathbf{y = ax^2 + bx + c}$

$h = -\frac{1}{2}$

At the y-intercept,

$3 = a(0)^2 + b(0) + c$

$c = 3$

When $x = 2$, $y = 0$,

$0 = a(2)^2 + b(2) + 3$

$0 = 4a + 2b + 3$ ------------ ❶

When $x = -3$, $y = 0$,

$0 = a(-3)^2 + b(-3) + 3$

$0 = 9a - 3b + 3$ ------------ ❷

❶×3 + ❷×2

$0 = 12a + 18a + 9 + 6$

$-30a = 15$

$a = -\frac{1}{2}$

$b = -\frac{1}{2}$

The function is:

$y = -\frac{1}{2}x^2 - \frac{1}{2}x + \cancel{6}\ 3$

$2y = -x^2 - x + \cancel{3}\ 6$

$y = -\frac{1}{2}\left(x + \frac{1}{2}\right)^2 + \frac{25}{8}$

$y = -\frac{1}{2}(x^2 + x - 3)$

$y = -\frac{1}{2}(x+3)(x-2)$

6. The polynomial $f(x) = x^3 + ax^2 + bx - 3$ is divisible by $(x - 3)$. When $f(x)$ is divided by $(x + 2)$, the remainder is 15. Find the value of a and b.

By Remainder theorem,

$f(3) = 0$

$(3)^3 + a(3)^2 + b(3) - 3 = 0$

$27 + 9a + 3b - 3 = 0$ $\quad 9a + 3b = -24$

$3a + b = -8$ -------------- ❶

By Remainder theorem,

$f(-2) = 0$

$(-2)^3 + a(-2)^2 + b(-2) - 3 = \cancel{0},\ 15$

$-8 + 4a - 2b - 3 = \cancel{0}\ 15$ $\quad (\cancel{8}4a - 2b = 26)$

$2a - b = 13$ --------------❷

❶ + ❷ $\quad 5a = 5$

$a = 1$

$b = -11$

7. If the graph of $\mathbf{y = mx^2 + 12x + 8}$ intersects the x-axis and **m is a <u>positive integer</u>**,

(a) find the FOUR possible values of m.

□ the graph intersects the x-axis

□ $\Delta > 0$

$(12)^2 - 4(m)(8) > 0$

$-32m > -144$

$m < 4.5$

□ m is a positive integer

□ $m = 1, 2, 3, 4$

the ~~biggest~~ largest possible positive integer value.

(b) when m is a **maximum**, find the **roots** of the equation $mx^2 + 12x + 8 = 0$.

$0 = 4x^2 + 12x + 8$

$0 = x^2 + 3x + 2$

$0 = (x + 2)(x + 1)$

$x = -2$ or $x = -1$

C. Challenging problem

The suggested time allocated for **Question 8 is 15 minutes**.

8. Given that the equation $(2k - 1)x^2 + (k - 3)x - 2 = 0$ where $k \neq \frac{1}{2}$,

(a) find the value of k if the **sum of roots equals the product of roots** of the equation.

$-b/a$

sum of roots $= \frac{-(k-3)}{2k-1} = \frac{-k+3}{2k-1}$

$\cancel{-\frac{b}{2a}}$ $\quad -\frac{b}{a}$

c/a

product of roots $= \frac{-2}{2k-1}$

$\cancel{\frac{-b^2+4ac}{4a}}$ $\quad \frac{c}{a}$.

□ sum of roots = product of roots

□ $\frac{-k+3}{2k-1} = \frac{-2}{2k-1}$

$(2k-1)(-k+3) = -2(2k-1)$

$-2k^2 + k \cancel{-} + 6k - 3 = -4k + 2$

$-2k^2 + 11k - 5 = 0$

$(-2k + 1)(k - 5) = 0$

$k = \frac{1}{2}$ or $k = 5$

□ $k \neq \frac{1}{2}$, □ $k = 5$

(b) By using the value of **k obtained in (a)**, find the **axis of symmetry**, **x-intercept(s)** and **y-intercept**.

$y = (10-1)x^2 + (5-3)x - 2$

$y = 9x^2 + 2x - 2$

At $y = 0$, $0 = 9x^2 + 2x - 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(9)(-2)}}{2(9)}$$

$$x = \frac{-1 \pm \sqrt{19}}{9}$$

□ x-intercepts $= \frac{-1+\sqrt{19}}{9}$ and $\frac{-1-\sqrt{19}}{9}$

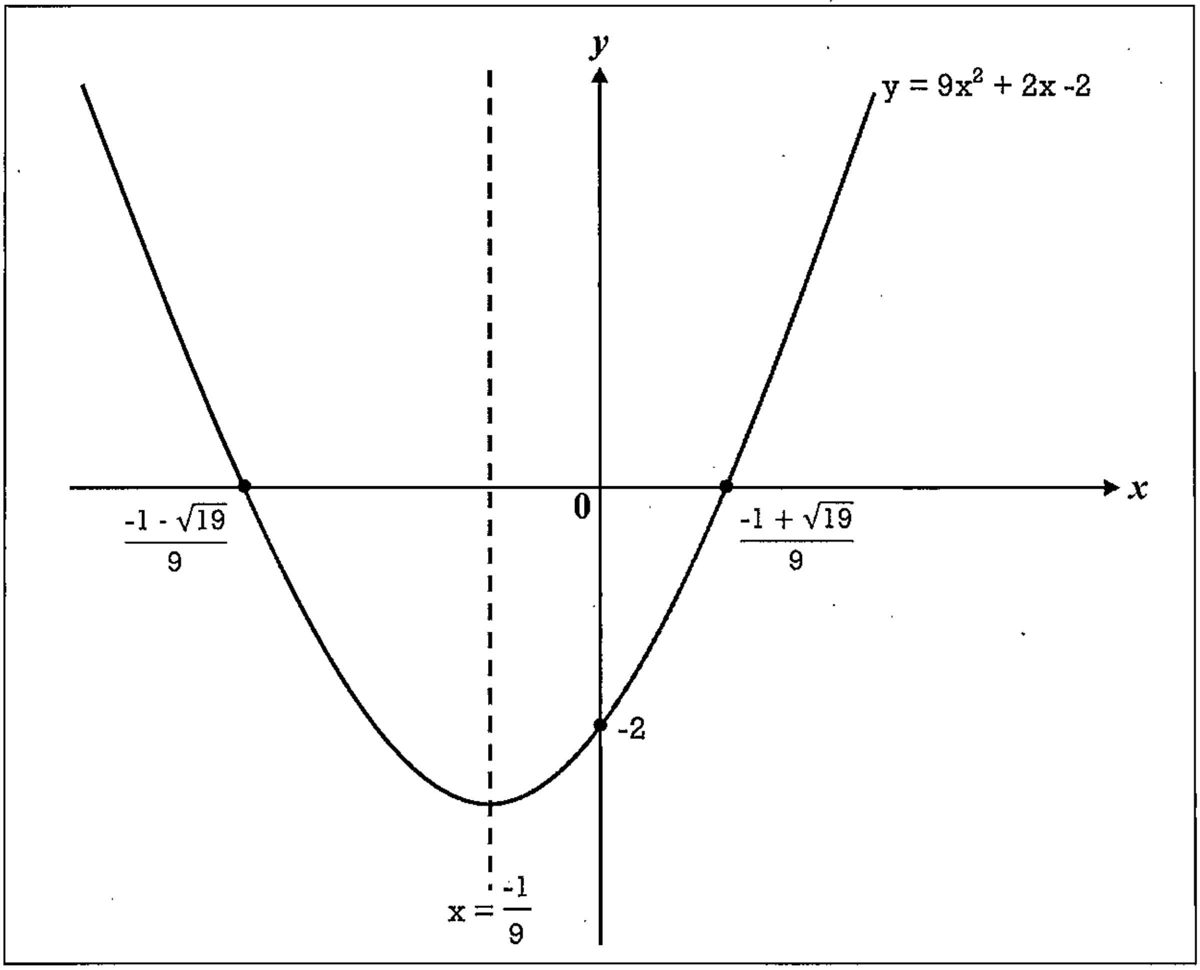
At $x = 0$, $y = -2$

□ y-intercept = -2

x-coordinate of the vertex $= \frac{-2}{2(9)} = \frac{-1}{9}$

□ The axis of symmetry is: $x = \frac{-1}{9}$ $\quad -\frac{b}{2a}$

(c) By using the results **obtained in (b)**, sketch the function $\mathbf{y = (2k - 1)x^2 + (k - 3)x - 2}$. The **axis of symmetry, x-intercept(s) and y-intercept** should be clearly shown on your graph.



D. Unfamiliar problems

The suggested time allocated for **<u>Question 9 is 15 minutes</u>**.

9. A courier company is responsible for delivering documents to Mainland China. Suppose the volume (in cm^3) of the rectangular carton used for delivery is given by $C(x) = x^3 - 180x^2 + 10700x - 210000$ (where $x > 70$).

MTR is the main means of transport used by that company. In order to reduce the cost, the cartons used must conform to the restrictions on the size of luggage carried by passengers on the MTR: **the sum of the length, width and height of the luggage <u>should not exceed 170 cm</u>, and the length of any side of the luggage <u>should not exceed 130 cm</u>.**

(a) Show that **$x - 70$** is a factor of $C(x)$.

$$
\begin{aligned}
C(70) &= (70)^3 - 180(70) + 10700(70) - 210000 \\
&= 343000 - 882000 + 749000 - 210000 \\
&= 0
\end{aligned}
$$

By Factor theorem,

$(x - 70)$ is the factor of $C(x)$.

(b) Factorize $C(x)$.

$$
\begin{aligned}
C(x) &= (x - 70)(x^2 - 110x + 3000) \\
&= (x - 70)(x - 50)(x - 60)
\end{aligned}
$$

$$
\begin{array}{r|l}
 & x^2 - 110x + 3000 \\
\hline
x - 70 & x^3 - 180x^2 + 10700x - 210000 \\
 & x^3 - 70x^2 \\
\hline
 & -110x^2 + 10700x - 210000 \\
 & -110x^2 + 7700x \\
\hline
 & 3000x - 210000 \\
 & 3000x - 210000 \\
\hline
\end{array}
$$

(c) (i) Find the value of $C(110)$.

$$
\begin{aligned}
C(110) &= (110)^3 - 180(110) + 10700(110) - 210000 \\
&= 1331000 - 2178000 + 1177000 - 210000 \\
&= 120000
\end{aligned}
$$

(ii) Using the results of **(b) and (c)(i)**, suggest the dimensions of a carton, with volume $120000 cm^3$, which conform to the restrictions on the size of luggage carried by passengers on the MTR.

☐ From (c)(i), the volume is equal to $120000\ cm^3$ when $x = 110$ cm

☐ the dimensions are $(110 - 70)(110 - 50)(110 - 60)$

$= (40\ cm)(60\ cm)(50\ cm)$

End of Assessment